

$$\begin{aligned}
\mathbf{B}_{tube} &= \nabla \times \left[ \frac{A(r, \theta)}{r \sin \theta} \hat{\phi} \right] + B_\phi(r, \theta) \hat{\phi} \\
A(r, \theta) &= \frac{1}{2} q a^2 B_t \exp \left[ -\frac{\varpi(r, \theta)}{a^2} \right] \\
B_\phi &= \frac{a B_t}{r \sin \theta} \exp \left[ -\frac{\varpi(r, \theta)}{a^2} \right]
\end{aligned}$$

where  $r$  is the radial distance from the origin;  $r_0(t)$ ,  $\theta$  is the polar angle from the polar axis,  $\hat{\phi}$  is the azimuthal direction,  $\varpi = (r^2 + R^2 - 2rR \sin^2 \theta)^{1/2}$  is the distance to the tube axis. Moreover, we have

$$\begin{aligned}
B_t &= 9 B_0 \\
B_0 &= 1.0 \\
q &= -1 \\
a &= 0, 1 L \\
L &= 0.0 \\
R &= 0.375
\end{aligned}$$

The Cartesian domain has the following sizes:

$$\begin{aligned}
n_X = 30 \quad L_X &= [0, 1.5] \\
n_Y = 20 \quad L_Y &= [0, 1.0] \\
n_Z = 25 \quad L_Z &= [0, 1.25]
\end{aligned}$$

We have set the following boundary conditions:

**BC<sub>X</sub>**: rigid walls;

**BC<sub>Y</sub>**: periodic;

**BC<sub>Z</sub>**: rigid walls;