

$$\begin{aligned}
B_r &= -\hat{B} \tanh(\xi) \\
B_\phi &= \sqrt{1-f} \frac{r_n}{r} \frac{B_0}{\cosh(\xi)} \\
B_x &= \epsilon \frac{r_n}{r} \left[\xi \tanh \xi \left(r_n \frac{\hat{B}'(r)}{\hat{B}} + \frac{r_n}{r} \right) - \frac{B_0'(r)}{B_0} \right] \\
B_0 &= \begin{cases} B_c \sqrt{c_0 + \left(1 - \frac{r-z_0}{r_y-z_0}\right) + \frac{c_1}{1+[(r-r_1)/L_1]^2}} & r \leq r_y \\ B_c \sqrt{c_0 + \frac{c_1}{1+[(r-r_1)/L_1]^2}} & r > r_y \end{cases} \\
p &= f \frac{B_0^2}{\cosh^2 \xi} \\
\xi &= \frac{r \hat{B} x}{\epsilon r_n^2} \\
r^2 &= y^2 + (z + z_0)^2 \\
\hat{B} &= B_0 \sqrt{f + (1-f) \frac{r_n^2}{r^2}} \\
f &= 0.2 \quad \text{and} \quad 0.4 \\
z_0 &= 10 \\
r_n &= 10 \\
r_y &= 40 \\
c_0 &= 0.5 \\
r_1 &= 22 \\
c_1 &= 0.5 \\
L_1 &= 3
\end{aligned}$$

The Cartesian domain has the following sizes:

$$\begin{aligned}
n_X &= 100 & L_X &= [-25.0, 25.0] \\
n_Y &= 160 & L_Y &= [-40.0, 40.0] \\
n_Z &= 80 & L_Z &= [0, 40.0]
\end{aligned}$$

We have set the following boundary conditions:

BC_X: rigid walls;

BC_Y: rigid walls;

BC_Z: open/outflow;